Poisson NL means: unsupervised non local means for Poisson noise

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Noise: **fluctuations** which corrupt a signal or an image,

- Poisson noise: due to **low-light conditions** when the number of collected photons is small,
  - ex: optical imagery, microscopy, astronomy.
Introduction to Poisson noise

- Noise: fluctuations which corrupt a signal or an image,
- Poisson noise: due to low-light conditions when the number of collected photons is small,
  - ex: optical imagery, microscopy, astronomy.

- Specificity of Poisson noise:
  - Signal-dependent,
  - True image and noise component not separable,

- Modeled by probability distributions.
Image denoising: find an estimation of the true image from the noisy image.

Noisy image

True image
Image denoising: find an estimation of the true image from the noisy image.

How to denoise an image?
- **Three main approaches**,  
- Lots of hybrid methods,  
- Majority designed for Gaussian noise.

- Variational / Markovian Approaches  
- Sparcifying transforms (wavelets, dictionnaries)  
- Non-local methods
Overview of denoising approaches

Image denoising: find an estimation of the true image from the noisy image.

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Overview of denoising approaches

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How to manage Poisson noise?
- Variance stabilisation:
  - Poisson noise $\rightarrow$ Gaussian noise,
- Method adaptation:
  - extend the method to Poisson noise,
- Our choice:
  - adapt the NL means for Poisson noise with a statistical point of view.
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Our contributions:
- new definition of the weights,
- automatic setting of the denoising parameters.
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1. Non-local estimation under Poisson noise

2. Automatic setting of the denoising parameters

3. Results of Poisson NL means
1 Non-local estimation under Poisson noise

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3 Results of Poisson NL means
Non-local estimation under Poisson noise

Non-local approach

- Local filters: **loss of resolution**,
- Non-local filters: **weights are defined from the image**,
- Weights are based on patch similarities.
Non-local estimation under Poisson noise

Non-local approach

- Local filters: loss of resolution,
- Non-local filters: weights are defined from the image,
- Weights are based on patch similarities.

[Buades et al., 2005]

Local approach

Weights map

Weighted average

Non-Local approach

Weights map

Weighted average

Search window

Noisy image

Patch comparison

Patch $B_s$

Patch $B_t$

Different patches $\Rightarrow$ low weights

Similar patches $\Rightarrow$ high weights
Non-local estimation under Poisson noise

Non-local approach

- Local filters: loss of resolution,
- Non-local filters: weights are defined from the image,
- Weights are based on patch similarities.

[Buades et al., 2005]

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Non-local estimation under Poisson noise

Non-local means [Buades et al., 2005]

- Define weights from the square differences between patches $B_s$ and $B_t$:

$$w_{s,t} = \exp \left(-\frac{\sigma}{\alpha}\right)$$

with $s+b$ and $t+b$ the $b$-th respective pixels in $B_s$ and $B_t$.

- Square differences: adapted for Gaussian noise,
- Which criterion for Poisson noise?
- How to set automatically the “optimal” parameter $\alpha$?
Weights have to select pixels with \textit{same true values},

Compare patches $\Leftrightarrow$ test the hypotheses that patches have:

$\mathcal{H}_0$ : same true values,
$\mathcal{H}_1$ : independent true values.

\[
P(\mathcal{H}_0|\text{image}_1, \text{image}_2) = \frac{P(\text{image}_1, \text{image}_2|\mathcal{H}_0)}{P(\text{image}_1, \text{image}_2|\mathcal{H}_1)} \times P(\mathcal{H}_0)
\]
Non-local estimation under Poisson noise

Patch-similarities: how to replace the square difference? [Deledalle et al., 2009]

- Weights have to select pixels with same true values,
- Compare patches ⇔ test the hypotheses that patches have:

\[ H_0 : \text{same true values}, \quad H_1 : \text{independent true values}. \]

\[
P(H_0|p_1, p_2) = \frac{P(p_1, p_2|H_0)}{P(p_1, p_2|H_1)} \times P(H_0)
\]

1. Similarity between noisy patches

- Based on detection theory, we propose to evaluate the generalized likelihood ratio (GLR) of both hypotheses given the noisy patches [Kay, 1998].

\[
- \log GLR(k_1, k_2) = k_1 \log k_1 + k_2 \log k_2 - (k_1 + k_2) \log \left( \frac{k_1 + k_2}{2} \right).
\]
### Patch-similarities: how to replace the square difference? [Deledalle et al., 2009]

- Weights have to select pixels with **same true values**, 

- Compare patches ⇔ test the hypotheses that patches have:

\[
\mathcal{H}_0 : \text{same true values}, \\
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\]

\[
P(\mathcal{H}_0|\hat{x}_1, \hat{x}_2) = \frac{P(\hat{x}_1, \hat{x}_2|\mathcal{H}_0)}{P(\hat{x}_1, \hat{x}_2|\mathcal{H}_1)} \times P(\mathcal{H}_0)
\]

### 2. Similarity between pre-filtered patches

- We propose to refine weights by using the similarity between pre-filtered patches.
  
  Idea motivated by [Polzehl et al., 2006, Brox et al., 2007, Goossens et al., 2008, Deledalle et al., 2009]

- A statistical test for the hypothesis \( \mathcal{H}_0 \) can be given by the **symmetrical Kullback-Leibler divergence**:

\[
D_{KL}(\hat{x}_1||\hat{x}_2) = (\hat{x}_1 - \hat{x}_2) \log \frac{\hat{x}_1}{\hat{x}_2}.
\]

- For Poisson noise, we obtain the following criterion:

\[
D_{KL}(\hat{\theta}_1||\hat{\theta}_2) = (\hat{\theta}_1 - \hat{\theta}_2) \log \frac{\hat{\theta}_1}{\hat{\theta}_2}.
\]
Non-local estimation under Poisson noise

Patch-similarities: how to replace the square difference? [Deledalle et al., 2009]

- Weights have to select pixels with same true values,
- Compare patches ⇔ test the hypotheses that patches have:

\[ P(H_0|\boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{P(\boldsymbol{x}_1, \boldsymbol{x}_2|H_0)}{P(\boldsymbol{x}_1, \boldsymbol{x}_2|H_1)} \times P(H_0) \]

\[ H_0 : \text{same true values}, \quad H_1 : \text{independent true values} \]

### Statistical tests

- Generalized likelihood ratio: \(-\log \text{GLR}\)
- Symmetrical Kullback-Leibler divergence: \(D_{KL}\)

### Weights

\[ w_{s,t} = \exp \left( -\frac{s}{\alpha} \right) \]

\[ w_{s,t} = \exp \left( -\frac{s}{\alpha} - \frac{t}{\beta} \right) \]

Better performances ⇐ Noisy + Pre-filtered patch similarities

Noisy patch similarities only
Non-local estimation under Poisson noise

Patch-similarities: how to replace the square difference? [Deledalle et al., 2009]

- Weights have to select pixels with **same true values**,
- Compare patches ⇔ test the hypotheses that patches have:

\[ \mathcal{H}_0 : \text{same true values}, \quad \mathcal{H}_1 : \text{independent true values}. \]

\[
P(\mathcal{H}_0 | \mathbf{x}_1, \mathbf{x}_2) = \frac{P(\mathbf{x}_1, \mathbf{x}_2 | \mathcal{H}_0)}{P(\mathbf{x}_1, \mathbf{x}_2 | \mathcal{H}_1)} \times P(\mathcal{H}_0)
\]

Next, how should we set the parameters \(\alpha\) and \(\beta\)?

**Statistical tests**

- **Generalized likelihood ratio**
  \[ -\log \text{GLR} \]
- **Symmetrical Kullback-Leibler divergence**
  \[ D_{KL} \]

Better performances ⇐

- **Noisy patch similarities only**
  \[ w_{s,t} = \exp \left( -\frac{\cdot}{\alpha} \right) \]
- **Noisy + Pre-filtered patch similarities**
  \[ w_{s,t} = \exp \left( -\frac{\cdot}{\alpha} - \frac{\cdot}{\beta} \right) \]
Non-local estimation under Poisson noise

How to choose the parameters?
(trade-off noisy/pre-filtered)
Non-local estimation under Poisson noise

How to choose the parameters? (trade-off noisy/pre-filtered)

Visually?

Blurry

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Non-local estimation under Poisson noise

How to choose the parameters? (trade-off noisy/pre-filtered)

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How to choose the parameters?
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Visually?
Non-local estimation under Poisson noise

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Visually?
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How to choose the parameters? (trade-off noisy/pre-filtered)

Visually?
Mean square error (MSE)?
Non-local estimation under Poisson noise

How to choose the parameters? (trade-off noisy/pre-filtered)

Visually?
Mean square error (MSE)?

How to estimate the MSE?
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Automatic setting of the denoising parameters

MSE estimators: unbiased risk estimators

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Gaussian</th>
<th>Poisson</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>SURE [Stein, 1981]</td>
<td>PURE [Chen, 1975]</td>
</tr>
<tr>
<td>Wavelet</td>
<td>SUREshrink [Donoho et al., 1995]</td>
<td>SURE-LET [Blu et al., 2007]</td>
</tr>
<tr>
<td>NL means</td>
<td>SURE based NL means [Van De Ville et al., 2009]</td>
<td>Poisson NL means</td>
</tr>
<tr>
<td></td>
<td>Local-SURE NL-means [Duval et al., 2010]</td>
<td></td>
</tr>
</tbody>
</table>

SURE: Stein’s Unbiaised Risk Estimator
PURE: Poisson Unbiaised Risk Estimator
Automatic setting of the denoising parameters

**PURE in Poisson NL means**
- Based on the same ideas as *SURE based NL means*:
  - PURE is obtained in closed-form for Poisson NL means,
  - with almost *same* computation time.

**Selection of the parameters**
- Optimum $\alpha$ and $\beta$ obtained iteratively using *Newton’s method*:
  \[
  \begin{pmatrix}
  \alpha_{n+1} \\
  \beta_{n+1}
  \end{pmatrix}
  =
  \begin{pmatrix}
  \alpha^n \\
  \beta^n
  \end{pmatrix}
  - H^{-1} \nabla
\]
  with $H$ the Hessian and $\nabla$ the gradient.
- The first and second order differentials of PURE are also obtained in closed-forms.

Find the best denoising level using similarities of noisy and pre-filtered patches!

MSE and PURE and their first and second order variations with respect to the parameters $\alpha$ and $\beta$
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Results of Poisson NL means

(a) Noisy image (16.59)  (b) NL means (26.30)  (c) Poisson-TV (26.42)

(b) NL means (26.30)  (c) Poisson-TV (26.42)

(e) PURE-LET (26.89)  (f) Poisson NL means (27.43)
Results of Poisson NL means

(a) Noisy image (16.59)

(b) NL means (26.30)

(c) Poisson-TV (26.42)

(d) PURE-LET (26.89)

(e) Our approach

(f) Poisson NL means (27.43)
Results of Poisson NL means

(a) Noisy image
(b) NL means
(c) Poisson-TV

(d) PURE-LET
(e) Poisson NL means

Cardiac mitochondrion,
Confocal fluorescence microscopy,
Image courtesy of Y. Tourneur.
Our contributions

1. New weights
   - Statistically grounded,
   - Based on noisy patches,
   - Based on pre-filtered patches.

2. Estimation of the risk
   - PURE for NL means,
   - Closed-form expression.

3. Minimization of the risk
   - First and second order derivates,
   - Newton's method.

Poisson NL means
- Good results,
- Unsupervised,
- Less than 10 iterations.
Our contributions

How to manage Poisson noise in NL means?

- Statistically grounded,
- Based on noisy patches,
- Based on pre-filtered patches.

Best setting of the parameters $\alpha, \beta$?

- PURE for NL means,
- Closed-form expression.

How to minimize PURE? optimal $\alpha, \beta$

- First and second order derivates,
- Newton’s method.

Poisson NL means
- Good results,
- Unsupervised,
- Less than 10 iterations.

Our perspectives

1. Extend to other noise model
   - Speckle noise,
   - Mixture of noise,
   - Vectorial images.

2. Unsupervised selection of:
   - Size/shape of the search window,
   - Size/shape of patches.
Questions?

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http://perso.telecom-paristech.fr/~deledall/poisson_nlmeans.php

→ More details and software available.
An intensity similarity measure in low-light conditions.
Lecture Notes in Computer Science, 3954:267.

Efficient Nonlocal Means for Denoising of Textural Patterns.
IEEE Transactions on Image Processing.

A Non-Local Algorithm for Image Denoising.

Poisson approximation for dependent trials.

Iterative Weighted Maximum Likelihood Denoising with Probabilistic Patch-Based Weights.

An improved non-local denoising algorithm.
In Proc. Int. Workshop on Local and Non-Local Approximation in Image Processing (LNLA’2008),
Lausanne, Switzerland.

Fundamentals of Statistical signal processing, Volume 2: Detection theory.
Prentice Hall PTR.
A variational approach to reconstructing images corrupted by Poisson noise.

Fast interscale wavelet denoising of Poisson-corrupted images.

Propagation-separation approach for local likelihood estimation.

Estimation of the mean of a multivariate normal distribution.

SURE-Based Non-Local Means.
Using the same model of weights for interferometric SAR data modeled by circular complexe Gaussian distributions ©DGA ©ONERA

(a) InSAR-SLC

(b) NL-InSAR
### Peppers (256 × 256)

<table>
<thead>
<tr>
<th>Method</th>
<th>PSNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noisy</td>
<td>3.14</td>
</tr>
<tr>
<td>MA filter</td>
<td>19.20</td>
</tr>
<tr>
<td>PURE-LET [Luisier et al., 2010]</td>
<td>19.33</td>
</tr>
<tr>
<td>NL means [Buades et al., 2005]</td>
<td>18.12</td>
</tr>
<tr>
<td>Poisson NL means</td>
<td><strong>19.90</strong></td>
</tr>
<tr>
<td><strong>α_{opt}</strong></td>
<td>(209)</td>
</tr>
<tr>
<td><strong>β_{opt}</strong></td>
<td>(0.72)</td>
</tr>
<tr>
<td><strong>#iterations</strong></td>
<td>(13.5)</td>
</tr>
</tbody>
</table>

### Cameraman (256 × 256)

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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Noisy</td>
<td>3.28</td>
</tr>
<tr>
<td>MA filter</td>
<td>18.71</td>
</tr>
<tr>
<td>PURE-LET [Luisier et al., 2010]</td>
<td>19.67</td>
</tr>
<tr>
<td>NL means [Buades et al., 2005]</td>
<td>18.17</td>
</tr>
<tr>
<td>Poisson NL means</td>
<td><strong>19.89</strong></td>
</tr>
<tr>
<td><strong>α_{opt}</strong></td>
<td>(62.1)</td>
</tr>
<tr>
<td><strong>β_{opt}</strong></td>
<td>(0.51)</td>
</tr>
<tr>
<td><strong>#iterations</strong></td>
<td>(11.0)</td>
</tr>
</tbody>
</table>

PSNR values averaged over ten realisations using different methods on images damaged by Poisson noise with different levels of degradation. The averaged optimal parameters and the averaged number of iterations of the proposed Poisson NL means are given.
Influence of the pre-filtered images

(a) Moving average (24.15)
(b) Poisson-TV (26.42)
(c) PURE-LET (26.89)
(d) PNLM+MA (27.43)
(e) PNLM+PTV (27.12)
(f) PNLM+P-LET (27.55)
Influence of the pre-filtered images

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Results with other images

(a) Noisy

(b) Poisson NL means
Results with other images

(a) Noisy

(b) Poisson NL means
Results with other images

(a) Noisy

(b) Poisson NL means
What is the difference between Gaussian and Poisson noise?

- **Gaussian noise:**
  - Constant variance,
  - True image + noise component.

- **Poisson noise:**
  - Signal-dependent,
  - True image and noise component not separable.
What the difference between Gaussian and Poisson noise?

**Gaussian noise**
- $v \in \mathbb{R}$: measured light intensity
- $u \in \mathbb{R}$: underlying light intensity
- $v | u \sim$ Gaussian distribution

**Poisson noise**
- $k \in \mathbb{N}$: number of collected photons.
- $\lambda \in \mathbb{R}^+$: underlying light intensity
- $k | \lambda \sim$ Poisson distribution
Automatic setting of parameters

Risk minimisation

Choose the parameters $\alpha$ et $\beta$ minimising the mean square error (MSE):

$$E \left[ \frac{1}{N} \| \lambda - \hat{\lambda} \|^2 \right] = \frac{1}{N} \sum_s \left( \lambda_s^2 + E \left[ \hat{\lambda}_s^2 \right] - E [\lambda_s \hat{\lambda}_s] \right)$$

- $\sum_s \lambda_s^2$ independent of the parameters,
- $\sum_s E \left[ \hat{\lambda}_s^2 \right]$ can be estimated from $\hat{\lambda}$,
- How to estimate $\sum_s E [\lambda_s \hat{\lambda}_s]$?

Poisson unbiased risk estimator (PURE) [Chen, 1975, Luisier et al., 2010]

- If $k$ is damaged by Poisson noise and $\hat{\lambda} = h(k)$ then

$$E \left[ \lambda_s \hat{\lambda}_s \right] = E [k_s \bar{\lambda}_s]$$

with $\bar{\lambda} = h(k)$ and $\bar{k}$ defined by

$$\bar{k}_t = \begin{cases} k_t - 1 & \text{if } t = s \\ k_t & \text{otherwise} \end{cases}$$

- PURE is given by:

$$R(\hat{\lambda}) = \frac{1}{N} \sum_s \left( \lambda_s^2 + \hat{\lambda}_s^2 - 2k_s \bar{\lambda}_s \right).$$
Automatic setting of parameters

Risk minimisation

Choose the parameters $\alpha$ et $\beta$ minimising the mean square error (MSE):

$$E \left[ \frac{1}{N} \| \lambda - \hat{\lambda} \|^2 \right] = \frac{1}{N} \sum_s \left( \lambda_s^2 + E \left[ \hat{\lambda}_s^2 \right] - E \left[ \lambda_s \hat{\lambda}_s \right] \right)$$

- $\sum_s \lambda_s^2$ independent of the parameters,
- $\sum_s E \left[ \hat{\lambda}_s^2 \right]$ can be estimated from $\hat{\lambda}$, 
- How to estimate $\sum_s E \left[ \lambda_s \hat{\lambda}_s \right]$?

PURE - Proof

Let be $k$ a r.v. following a Poisson distribution and $h(.)$ a function:

$$E \left[ kh(k - 1) \right] = \sum_{k=1}^{\infty} kh(k - 1) \frac{\lambda^k e^{-\lambda}}{k!}$$

$$= \lambda \sum_{k=1}^{\infty} h(k - 1) \frac{\lambda^{k-1} e^{-\lambda}}{(k - 1)!}$$

$$= E \left[ \lambda h(k) \right]$$