A Non-Local Approach for SAR and Interferometric SAR Denoising

Charles Deledalle\textsuperscript{1}, Florence Tupin\textsuperscript{1}, Loïc Denis\textsuperscript{2}

\textsuperscript{1} Institut Telecom, Telecom ParisTech, CNRS LTCI, Paris, France
\textsuperscript{2} Observatoire de Lyon, CNRS CRAL, UCBL, ENS de Lyon, Université de Lyon, Lyon, France

July 27, 2010
Why non-local methods?

(a) Noisy image  
(b) Non-local means

Noise reduction + resolution preservation
Why non-local methods?

(a) Noisy image
(b) Non-local means

Noise reduction + resolution preservation

- Goal: to adapt non-local methods to SAR and InSAR data,
- Method: take into account the statistics of SAR and InSAR data,
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1 Non-local means (NL means)

2 Non-local estimation for SAR data
   - Weighted maximum likelihood
   - Setting of the weights

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   - Results on SAR data
   - Extension and results for InSAR data
1. Non-local means (NL means)

2. Non-local estimation for SAR data
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3. Results of NL-SAR and NL-InSAR
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Non-local means (NL means) (1/2)

**Background**

- Local filters (e.g. boxcar, Gaussian filter...) ⇒ resolution loss,
- To combine *similar pixels* instead of neighboring pixels.

\[
\hat{u}_s = \frac{1}{Z} \sum_t e^{-\frac{|s-t|^2}{\rho^2}} v_t
\]

Gaussian filter

\[
\hat{u}_s = \frac{1}{Z} \sum_t e^{-\frac{\text{sim}(s,t)}{h^2}} v_t
\]

[Yaroslavsky, 1985]
Non-local means (NL means) (1/2)

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[Yaroslavsky, 1985]

Non-local means [Buades et al., 2005]

- Similarity evaluated using square patches \(\Delta_s\) and \(\Delta_t\) centered on \(s\) and \(t\),
- Consider the redundant structure of images.
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- Consider the redundant structure of images.
- Hyp. 1: similar neighborhood ⇒ same central pixel,
- Hyp. 2: each patch is redundant (can be found many times).
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\]
Similarity criterion [Buades et al., 2005]

- **Euclidean distance:**
  \[
  \text{sim}(s, t) = \sum_k |v_{s,k} - v_{t,k}|^2
  \]

  with \( k \) the \( k \)-th respective pixel of \( \Delta_s \) and \( \Delta_t \).
Non-local means (NL means) (2/2)

**Similarity criterion** [Buades et al., 2005]

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  with \(k\) the \(k\)-th respective pixel of \(\Delta_s\) and \(\Delta_t\).

- Hyp. 1: similar neighborhood \(\Rightarrow\) same central pixel,
- Hyp. 2: each patch is redundant (can be found many times),
- Hyp. 3: the noise is additive, white and Gaussian.
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Context and notations

- **Searched data:** $R_s$ the reflectivity,
- **Noise model:** $p(\cdot|R_s)$ a Rayleigh distribution,
- **Noisy observations:** $A_s$ the amplitude such that $A_s \sim p(\cdot|R_s)$,
- **To denoise:** to search an estimate $\hat{R}_s$ of $R_s$. 

Weighted maximum likelihood for SAR data
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From weighted average to weighted maximum likelihood

- NL means perform a weighted average of noisy pixel values,
- For SAR data, we suggest to use a weighted maximum likelihood:

\[
\hat{R}_s = \arg \max_R \sum_t w(s, t) \log p(A_t | R) = \frac{\sum_t w(s, t) A_t^2}{\sum_t w(s, t)}
\]

where \( w(s, t) \) is a weight approaching the indicator function of the set of redundant pixels (i.e. with i.i.d. values): \( \{s, t | R_s = R_t\} \).
Weighted maximum likelihood for SAR data

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**From weighted average to weighted maximum likelihood**

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where $w(s, t)$ is a weight approaching the indicator function of the set of redundant pixels (i.e. with i.i.d values): $\{s, t| R_s = R_t\}$.

- Choice of $w(s, t)$: oriented, adaptive or **non-local** neighborhoods.
Weighted maximum likelihood for SAR data

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Example (Refined Lee - oriented neighborhood [Lee, 1981])

- Redundant pixels are located in one of these eight neighborhoods:

  ![Image extracted from [Lee, 1981]](image)
Weighted maximum likelihood for SAR data

Context and notations

- Searched data: $R_s$ the reflectivity,
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Example (IDAN - adaptive neighborhood [Vasile et al., 2006])

- Redundant pixels are located in an adaptive neighborhood (obtained by region growing algorithm):

  ![Image extracted from [Vasile et al., 2006]]
Weighted maximum likelihood for SAR data

Context and notations

- **Searched data:** $R_s$ the reflectivity,
- **Noise model:** $p(.|R_s)$ a Rayleigh distribution,
- **Noisy observations:** $A_s$ the amplitude such that $A_s \sim p(.|R_s)$,
- **To denoise:** to search an estimate $\hat{R}_s$ of $R_s$.

Example (NL means - non-local neighborhood)

- Redundant pixels are located anywhere in the image:

![Image](image extracted from [Buades et al., 2005])
We search weights $w(s, t)$ such that:

- $w(s, t)$ is high if $R_s = R_t$,
- $w(s, t)$ is low if $R_s \neq R_t$. 
Setting of the weights for SAR data (1/3)

- We search weights $w(s, t)$ such that:
  - $w(s, t)$ is high if $R_s = R_t$,
  - $w(s, t)$ is low if $R_s \neq R_t$.

Statistical similarity between noisy patches [Deledalle et al., 2009]

- Using the hypothesis of NL means:
  - Patches $\Delta_s$ and $\Delta_t$ similar $\Rightarrow$ central values $s$ and $t$ close.
- Weights are defined as follows:

\[
w(s, t) = p(\Delta_{s,k} = \Delta_{t,k} | A_{s,k}, A_{t,k})^{1/h} = \prod_k p(R_{s,k} = R_{t,k} | A_{s,k}, A_{t,k})^{1/h}
\]

with $p(R_{s,k} = R_{t,k} | A_{s,k}, A_{t,k})$ statistical similarity

$h$ regularization parameter.
Bayesian decomposition

\[ p(R_1 = R_2 | A_1, A_2) \propto p(A_1, A_2 | R_1 = R_2) \times p(R_1 = R_2) \]

- similarity likelihood
- a priori similarity
Setting of the weights for SAR data (2/3)

Bayesian decomposition

\[ p(R_1 = R_2 | A_1, A_2) \propto p(A_1, A_2 | R_1 = R_2) \times p(R_1 = R_2) \]

- similarity likelihood
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Similarity likelihood

\[
p(A_1, A_2 | R_1 = R_2) = \frac{\int p(A_1 | R_1 = R)p(A_2 | R_2 = R)p(R_1 = R)p(R_2 = R) \, dR}{\int p(R_1 = R)p(R_2 = R) \, dR}
\]

- Since \( p(R_1 = R) \) is unknown, we define:

\[
p(A_1, A_2 | R_1 = R_2) \triangleq \int p(A_1 | R_1 = R)p(A_2 | R_2 = R) \, dR
\]

- The corresponding similarity criterion is then:

\[
- \log p(A_1, A_2 | R_1 = R_2) \propto \log \left( \frac{A_1}{A_2} + \frac{A_2}{A_1} \right)
\]
Setting of the weights for SAR data (2/3)

Bayesian decomposition

\[ p(R_1 = R_2 | A_1, A_2) \propto p(A_1, A_2 | R_1 = R_2) \times p(R_1 = R_2) \]

- similarity likelihood
- a priori similarity

Refined similarity

- Refining the weights is necessary when the signal to noise ratio is low.
- Using a pre-estimate \( \hat{R} \) at pixel 1 and 2 provides two estimations of the noise models:
  \[ p(\cdot | \hat{R}_1) \quad \text{and} \quad p(\cdot | \hat{R}_2). \]
- Assuming \( p(R_1 = R_2) \) depends on the proximity of \( p(\cdot | \hat{R}_1) \) to \( p(\cdot | \hat{R}_2) \), then we define
  \[ p(R_1 = R_2) \triangleq \exp\left(-\frac{SD_{KL}(\hat{R}_1 || \hat{R}_2)}{T}\right) \]
  with \( SD_{KL}(\hat{R}_1, \hat{R}_2) \propto \frac{(\hat{R}_1 - \hat{R}_2)^2}{\hat{R}_1 \hat{R}_2} \)
- The Kullback-Leibler divergence provides a statistical test of the hypothesis \( R_1 = R_2 \) [Polzehl and Spokoiny, 2006]
Iterative scheme

The refined similarity involves an iterative scheme in two steps:

1. Estimate the weights from $A$ and $\hat{R}^{i-1}$:

$$- \log w(s, t) \leftarrow \frac{1}{h} \sum \log \left( \frac{A_1}{A_2} + \frac{A_2}{A_1} \right) + \frac{1}{T} \sum \frac{(\hat{R}_1^{i-1} - \hat{R}_2^{i-1})^2}{\hat{R}_1^{i-1} \hat{R}_2^{i-1}},$$

2. Maximize the weighted likelihood:

$$\hat{R}_s^i \leftarrow \frac{\sum_t w(s, t) A_t^2}{\sum_t w(s, t)}$$

The procedure converges in about ten iterations.

Scheme of the iterative filtering process
Non-local means (NL means)

Non-local estimation for SAR data
- Weighted maximum likelihood
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Results of NL-SAR and NL-InSAR
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Results of NL-SAR (Toulouse, France, TerraSAR-X ©DLR)

(a) SAR-SLC

(b) Refined Lee [Lee et al., 2003]
Results of NL-SAR (Toulouse, France, TerraSAR-X ©DLR)

(a) SAR-SLC

(b) WinSAR [Achim et al., 2003]
Results of NL-SAR (Toulouse, France, TerraSAR-X ©DLR)

(a) SAR-SLC

(b) NL-SAR
Extension to InSAR data

Statistical model of InSAR data

\[
p(z, z' | \Sigma) = \frac{1}{\pi^2 \text{det}(\Sigma)} \times \exp \left[ - (z^* z') \Sigma^{-1} \begin{pmatrix} z \\ z' \end{pmatrix} \right]
\]

\[
\Sigma = \mathbb{E} \left\{ \begin{pmatrix} z \\ z' \end{pmatrix} (z^* z') \right\} = \begin{pmatrix} R & R D e^{i\beta} \\ R D e^{-i\beta} & R \end{pmatrix}
\]

- Observations:
  - \( z \) and \( z' \) two co-registered single-look complex values.

- Parameters to estimate:
  - \( R \) the reflectivity,
  - \( \beta \) the true phase difference, and
  - \( D \) the coherence.

- Challenge: vectorial data with channels of different nature
  - \( z, z' \in \mathbb{C} \),
  - \( R \in \mathbb{R}^+ \),
  - \( \beta \in [-\pi, \pi] \) (wrapped data),
  - \( D \in [0, 1] \).
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Suitable similarity criteria can be derived from this joint distribution.
Extension to InSAR data

Closed-form expressions for InSAR data

- The WMLE for InSAR data can be expressed as:

\[ \hat{R}_s = \frac{a}{N}, \quad \hat{\beta}_s = -\text{arg} \, x, \quad \hat{D}_s = \frac{|x|}{a} \]

\[ a = \sum_tw(s, t) \frac{|z_t|^2 + |z_t'|^2}{2}, \]

with \( x = \sum_tw(s, t) z_t z'_t \), \( N = \sum_t w(s, t) \).

- The similarity between noisy data is given by:

\[
\sqrt{\frac{C}{B}} \left( \frac{A + B}{A} \sqrt{\frac{B}{A - B}} - \arcsin \sqrt{\frac{B}{A}} \right) = \frac{4}{\pi} \left[ (1 - \hat{D}_1 \hat{D}_2 \cos(\hat{\beta}_1 - \hat{\beta}_2)) \left( \frac{\hat{R}_1}{\hat{R}_2(1 - \hat{D}_2^2)} + \frac{\hat{R}_2}{\hat{R}_1(1 - \hat{D}_1^2)} \right) - 2 \right]
\]

with \( A = \left( |z_1|^2 + |z'_1|^2 + |z_2|^2 + |z'_2|^2 \right)^2 \), \( B = 4 |z_1 z'_1 + z_2 z'_2|^2 \), and \( C = |z_1 z'_1 z_2 z'_2| \).
(a) InSAR-SLC

(b) Refined Lee [Lee et al., 2003]
(a) InSAR-SLC

(b) IDAN [Vasile et al., 2006]
Results of NL-InSAR (Toulouse, France, RAMSES ©DGA ©ONERA)

(a) InSAR-SLC

(b) NL-InSAR
### Numerical results of NL-SAR and NL-InSAR on a resolution test-pattern

<table>
<thead>
<tr>
<th>Channel</th>
<th>$R_{SAR}$</th>
<th>$R_{InSAR}$</th>
<th>$\beta$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLC</td>
<td>-4.42</td>
<td>-2.75</td>
<td>3.36</td>
<td>-1.19</td>
</tr>
<tr>
<td>Refined Lee [Lee et al., 2003]</td>
<td>5.47</td>
<td>6.23</td>
<td>9.12</td>
<td>2.03</td>
</tr>
<tr>
<td>WinSAR [Achim et al., 2003]</td>
<td>5.49</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>IDAN [Vasile et al., 2006]</td>
<td>–</td>
<td>5.00</td>
<td>7.88</td>
<td>0.33</td>
</tr>
<tr>
<td><strong>NL-(In)SAR</strong></td>
<td><strong>7.46</strong></td>
<td><strong>9.02</strong></td>
<td><strong>13.04</strong></td>
<td><strong>6.92</strong></td>
</tr>
</tbody>
</table>

**SNR values of estimated SAR and InSAR images using different estimators**

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**Results of NL-InSAR on the resolution test-pattern**
Conclusion

- We proposed an efficient estimator of SAR and InSAR data based on non-local approaches,
- The idea is to search iteratively the most suitable pixels to combine,
- Similarity criteria:
  - joint similarity between the noisy observations of surrounding patches, and
  - joint similarity between the pre-filtered data of surrounding patches.
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- Results on simulated, Ramses and TerraSAR-X data:
  - Good noise reduction without significant loss of resolution,
  - Closer to the noise-free image than state-of-the-art estimators.
We proposed an efficient estimator of SAR and InSAR data based on non-local approaches,

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More details in:


website: http://perso.telecom-paristech.fr/~deledall
NL-SAR software available
SAR image denoising via Bayesian wavelet shrinkage based on heavy-tailed modeling.

A Non-Local Algorithm for Image Denoising.

Iterative Weighted Maximum Likelihood Denoising with Probabilistic Patch-Based Weights.

Speckle phenomena in optics: theory and applications.

Refined filtering of image noise using local statistics.

Speckle filtering and coherence estimation of polarimetric SAR interferometry data for forest applications.

Propagation-separation approach for local likelihood estimation.

Intensity-driven adaptive-neighborhood technique for polarimetric and interferometric SAR parameters estimation.

Digital Picture Processing.
Springer-Verlag New York, Inc. Secaucus, NJ, USA.
Results of NL-InSAR (Saint-Gervais, France, TerraSAR-X ©DLR)

(a) InSAR-SLC

(b) NL-InSAR
Distributions of SAR and InSAR data

- Distribution of SAR data, Rayleigh distribution:
  \[
p(A|R) = \frac{2A}{R} \exp \left( -\frac{A^2}{R} \right)
  \]

- Distribution of InSAR data [Goodman, 2006]:
  \[
p(z, z'|\Sigma) = \frac{1}{\pi^2 \det(\Sigma)} \times \exp \left[ - (z^* z') \Sigma^{-1} \left( \begin{array}{c} z \\ z' \end{array} \right) \right] = \frac{R}{\sqrt{RR'} D e^{j\beta}}
  \]

  which is equivalent to:
  \[
p(A, A', \Delta \phi|R, D, \beta) = \frac{2AA'}{\pi R^2 (1 - D^2)} \times \exp \left( - \frac{A^2 + A'^2 - 2DA A'}{R(1 - D^2)} \cos(\Delta \phi - \beta) \right).
  \]

  with \( z = A^{i\phi} \), \( z' = A'^{i\phi'} \) and \( \Delta \phi = \phi - \phi' \).
Closed-form expressions of WMLE for SAR and InSAR data

- WMLE for SAR data:
  \[
  \hat{R}_s = \frac{\sum_t w(s, t) A_t^2}{\sum_t w(s, t)}
  \]

- WMLE for InSAR data:
  \[
  \hat{R}_s = \frac{a}{N},
  \]
  \[
  \hat{\beta}_s = - \arg x,
  \]
  \[
  \hat{D}_s = \frac{|x|}{a}
  \]

with
\[
a = \sum_t w(s, t) \frac{|z_t|^2 + |z_t'|^2}{2},
\]
\[
x = \sum_t w(s, t) z_t z_t'^*,
\]
\[
N = \sum_t w(s, t).
\]
Closed-form expressions of similarities for SAR and InSAR data

<table>
<thead>
<tr>
<th>Similarity between . . .</th>
<th>SAR</th>
<th>InSAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>. . . noisy data</td>
<td>( \log \left( \frac{A_1}{A_2} + \frac{A_2}{A_1} \right) )</td>
<td>( - \log \left[ \sqrt{\frac{C^3}{B}} \left( \frac{A + B}{A} \sqrt{\frac{B}{A - B}} - \arcsin \sqrt{\frac{B}{A}} \right) \right] )</td>
</tr>
<tr>
<td>. . . pre-filtered data</td>
<td>( \frac{(\hat{R}_1 - \hat{R}_2)^2}{\hat{R}_1 \hat{R}_2} )</td>
<td>( \frac{4}{\pi} \left[ (1 - \hat{D}_1 \hat{D}_2 \cos(\beta_1 - \beta_2)) \left( \frac{\hat{R}_1}{\hat{R}_2 (1 - \hat{D}_2^2)} + \frac{\hat{R}_2}{\hat{R}_1 (1 - \hat{D}_1^2)} \right) - 2 \right] )</td>
</tr>
</tbody>
</table>

with

\( A = \left( |z_1|^2 + |z'_1|^2 + |z_2|^2 + |z'_2|^2 \right)^2 \)

\( B = 4 \left| z_1 z'_1 + z_2 z'_2 \right|^2 \)

\( C = \left| z_1 z'_1 z_2 z'_2 \right| \)